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PRELIMINARY SYSTEMS ANALYSIS FOR HOVERING VEHICLES

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ABSTRACT. A rapid method for calculating the required system weight ratio for a rocket-powered hovering vehicle is presented. Two cases are considered: the first for vehicles operating in a regime where specific impulse is essentially constant, and the second for vehicles operating where the variation of specific impulse with thrust must be considered. An equation is also developed that yields total system weight as a function of payload weight, system weight ratio, and propulsion unit weight ratio.

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NOMENCLATURE

c^*	Characteristic velocity, ft/sec
C_F	Thrust coefficient
F	Thrust, lb
g	Gravitational acceleration, 32.2 ft/sec^2
I_{sp}	Specific impulse, lb-sec/lb
K_1	(See text)
m	Instantaneous weight, lb
m_e	Weight of exhaust products, lb
m_p	Propellant weight, lb
m_{pl}	Payload weight, lb
m_{po}	Initial propulsion unit weight, lb
m_{sb}	System burnt weight, lb
m_{so}	Initial system weight, lb
P_a	Ambient atmospheric pressure, lb/in^2
P_c	Chamber pressure, lb/in^2
P_e	Nozzle exit pressure, lb/in^2
t	Instantaneous time, sec
t_b	~~Hovering~~ time, sec
v_e	Effective exhaust velocity, ft/sec
ϵ	Nozzle expansion ratio
γ	Specific heat ratio
λ_s	System weight ratio, (m_{sb}/m_{so})
λ_p	Propulsion unit weight ratio (m_p/m_{po})

INTRODUCTION

Various potential applications exist for a vehicle capable of hovering at a given point above the surface of the earth. Observation of earth and space phenomena, both natural and man-made, would be possible with such a vehicle.

Studies of launching vehicles, hovering propulsion systems selection, and mission parameters, such as time and range to station, have been made elsewhere (Ref. 1); therefore, they are not covered in this report.

Examination of propulsion system requirements for a hovering mission indicates that total system weight is extremely sensitive to the obtainable propulsion unit weight ratio and the required hovering time. To simplify systems study, it is desirable to express the required weight ratios as functions of the parameters of interest; i.e., hovering time, propellant capacity, and motor performance. In order to accomplish this, two cases will be examined. Case I will assume the specific impulse to be constant over the thrust range, as would be the situation if the hovering point were above the sensible atmosphere. Case II takes into account the variation of specific impulse with thrust level in situations where ambient pressure at the hovering altitude cannot be ignored. Assumptions made in both cases are that the characteristic velocity (c^*) remains constant as the thrust varies, and that nozzle geometry remains fixed.

It can also be shown that the total vehicle weight for a given hovering time is a direct function of payload weight and the two weight ratios of interest; i.e., the ratio of burnt system weight to initial system weight, and the propellant weight to the initial propulsion unit weight ratio.

DERIVATION OF SYSTEM WEIGHT RATIO EQUATION

CASE I

For a vehicle in equilibrium, a force balance yields the equation

$$\frac{d\,(m_e v_e)}{dt} - mg = 0 \tag{1}$$

Expansion gives

$$v_e \frac{dm_e}{dt} + m_e \frac{dv_e}{dt} = mg \tag{2}$$

In this case,

$$\frac{dv_e}{dt} = 0$$

and

$$v_e = I_{sp} g \tag{3}$$

Also

$$m_e = m_{so} - m \tag{4}$$

hence

$$I_{sp} \left(\cancelto{0}{\frac{dm_{so}}{dt}} - \frac{dm}{dt} \right) = m$$

or

$$\frac{dm}{m} = - \frac{1}{I_{sp}} \; dt \tag{5}$$

Integration over the operating time yields

$$\ln \frac{m_{sb}}{m_{so}} = - \frac{t_b}{I_{sp}} \tag{6}$$

or

$$\frac{m_{sb}}{m_{so}} = e^{-\frac{t_b}{I_{sp}}} \tag{7}$$

Thus, substitution of desired hovering time and obtainable specific impulse will furnish the required system weight ratio.

CASE II

Equation (2) is also applicable to this case.

However,

$$\frac{dv_e}{dt} \neq 0$$

Substituting values for m_e and v_e from equations (3) and (4), respectively,

$$(m_{so} - m) \frac{dI_{sp}}{dt} - I_{sp} \frac{dm}{dt} = m \tag{8}$$

also

$$\frac{dI_{sp}}{dt} = \frac{dI_{sp}}{dm} \frac{dm}{dt} \tag{9}$$

Substituting (9) in (8) and rearranging,

$$\left[(m_{so} - m) \frac{dI_{sp}}{dm} - I_{sp}\right] \frac{dm}{m} = dt \tag{10}$$

The solution of (10) requires that I_{sp} be expressed as a function of m. The two applicable equations (Ref. 2) are:

$$I_{sp} = \frac{c^* C_F}{g} \tag{11}$$

and

$$C_F = \sqrt{\frac{2\gamma^2}{\gamma-1} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \frac{P_e - P_a}{P_c} \epsilon \tag{12}$$

Equation (12) can be written as

$$C_F = \epsilon \left(K_1 - \frac{P_a}{P_c}\right) \tag{13}$$

where

$$K_1 = \frac{1}{\epsilon} \left[\sqrt{\frac{2\gamma^2}{\gamma-1} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}\right] + \frac{P_e}{P_c} \tag{14}$$

The ratio $\frac{P_e}{P_c}$ is a function of only γ and ϵ, as shown by the following: (Ref. 2)

$$\frac{1}{\epsilon} = \left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}} \left(\frac{P_e}{P_c}\right)^{\frac{1}{\gamma}} \sqrt{\frac{\gamma+1}{\gamma-1}\left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{15}$$

Computer solution of equations (14) and (15) will then yield K_1 as a function of ϵ and γ. These results are plotted in Fig. 1.

In addition, it can be shown that, for constant nozzle geometry, the ratio of thrust to chamber pressure is approximately constant over a reasonable thrust range. Therefore,

$$\frac{F}{P_c} = \frac{F(max)}{P_{c(max)}} \tag{16}$$

However,

$$F = m$$

$$F_{(max)} = m_{so}$$

Hence

$$P_c = \frac{m}{m_{so}} P_{c(max)} \tag{17}$$

Substituting (17) into (14),

$$C_F = \epsilon \left(K_1 - \frac{P_a}{P_{c(max)}} \frac{m_{so}}{m}\right) \tag{18}$$

Thus

$$I_{sp} = Q \left(K_1 - R \frac{m_{so}}{m}\right) \tag{19}$$

where

$$Q = \frac{c^* \epsilon}{g}$$

$$R = \frac{P_a}{P_{c(max)}}$$

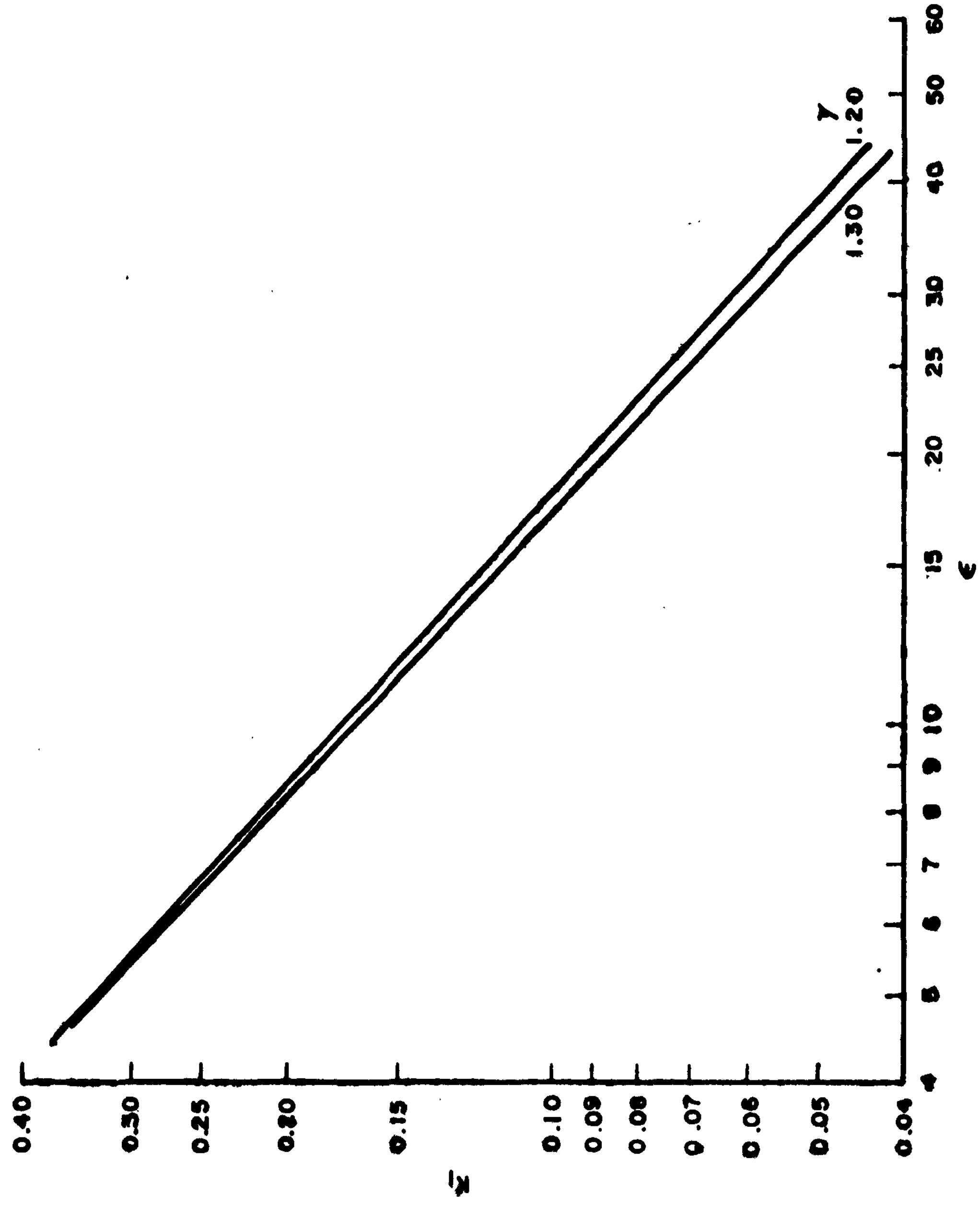

FIG. 1. Factor K_1 vs. Expansion Ratio ϵ.

Then

$$\frac{dI_{sp}}{dm} = QR\,\frac{m_{so}}{m^2} \tag{20}$$

which gives us the required expressions involving I_{sp} as a function of m.

Returning to equation (10), and substituting the appropriate expressions

$$\left[\left(m_{so} - m\right) \quad QR\,\frac{m_{so}}{m^2} - Q\left(K_1 - R\,\frac{m_{so}}{m}\right)\right]\frac{dm}{m} = dt \tag{21}$$

Expanding and collecting terms

$$\left(\frac{QRm_{so}^{2}}{m^3} - \frac{QK_1}{m}\right) dm = dt \tag{22}$$

Integration, and substitution of the expressions for Q and R, finally yields

$$K_1 \ln \frac{m_{sb}}{m_{so}} + \frac{P_a}{2P_{c(max)}}\left[\frac{1}{\left(\frac{m_{sb}}{m_{so}}\right)^2} - 1\right] = -\frac{gt_b}{c^*\epsilon} \tag{23}$$

Thus an expression is obtained for system weight ratio as a function of hovering time, local atmospheric pressure and gravitational acceleration, maximum chamber pressure, characteristic velocity, and nozzle expansion ratio--all independent variables that may be chosen as desired.

CALCULATION OF VEHICLE WEIGHT

With the system weight ratio defined, total vehicle weight as a function of payload weight can be found.

The following parameters are defined:

$$\lambda_s = \frac{m_{sb}}{m_{so}}$$

$$\lambda_p = \frac{m_p}{m_{po}}$$

Then, initial system weight is given by

$$m_{so} = \frac{\lambda_p}{(\lambda_s + \lambda_p) - 1} \, m_{pl} \tag{24}$$

Examination of this equation would indicate at first glance that a real system is possible only when

$$\lambda_s + \lambda_p > 1$$

However, for cases where this sum is less than one, a real value for m_{so} may be obtained by considering a portion of the propulsion unit inert weight as payload. This has the result of increasing the propulsion unit weight ratio λ_p to a higher value such that $(\lambda_s + \lambda_p)$ is greater than one. Necessity of resorting to this manipulation generally indicates that a large vehicle will be required to perform the hovering mission.

VALIDITY OF RESULTS

One non-rigorous assumption is made in the development of the Case II equation, this being the assumption that the ratio of thrust to chamber pressure is a constant over the thrust range. Previous studies indicate that over the range of interest--approximately 20% to 100% of maximum thrust--the ratio is constant within ±10%. The effect of this variation on the overall result is small, however, being of the order of 1 to 2%.

The area where the Case II equation is useful is obviously at lower altitudes, where the variation of I_{sp} with thrust level is considerable. A comparison of the results obtained from the two equations as a function of altitude for a typical system indicates that significant differences begin to occur below an altitude of about 100,000 feet, and that Case II defines the required weight ratio more accurately within this range.

SUMMARY OF EQUATIONS

CASE I

$$\frac{m_{sb}}{m_{so}} = e^{-\frac{t_b}{I_{sp}}}$$

Range of interest - above 100, 000 feet altitude

CASE II

$$K_1 \ln \frac{m_{sb}}{m_{so}} + \frac{P_a}{2P_{c(max)}} \left[\frac{1}{\left(\frac{m_{sb}}{m_{so}}\right)^a} - 1 \right] = - \frac{gt_b}{c^* \epsilon}$$

Range of interest - below 100, 000 feet altitude

WEIGHT CALCULATION

$$m_{so} = \frac{\lambda_p}{(\lambda_s + \lambda_p) - 1} \, m_{pl}$$

EXAMPLE

A vehicle is required to hover at an altitude of 50, 000 feet for 4 minutes carrying a payload of 250 pounds. Assume the following:

$\lambda_p = 0.800$	$P_{c(max)} = 200$ psia
$c^* = 5,000$ ft/sec	$\epsilon = 10$
$\gamma = 1.20$	

The total vehicle weight is desired. Atmospheric pressure at 50, 000 feet altitude is 1. 69 psia (Ref. 2). From Fig. 1

$$K_1 = 0.174$$

Then, substituting known values into equation (23)

$$0.174 \ln \frac{m_{sb}}{m_{so}} + 0.004 \left[\frac{1}{\left(\frac{m_{sb}}{m_{so}}\right)^a} - 1 \right] = - 0.154$$

Solving for $\frac{m_{sb}}{m_{so}}$ yields

$$\frac{m_{sb}}{m_{so}} = \lambda_s = 0.350$$

then, from equation (24)

$$m_{so} = \frac{0.800}{(0.350 + 0.800) - 1} (250) = \underline{1330 \text{ lb}}$$

By contrast, the value of total vehicle weight obtained by assuming a constant average I_{sp} of 240 seconds is

$$m_{so} = 1190 \text{ lb}$$

which indicates the large discrepancy encountered with the Case I equation as the operating altitude decreases.

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ABSTRACT CARD

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